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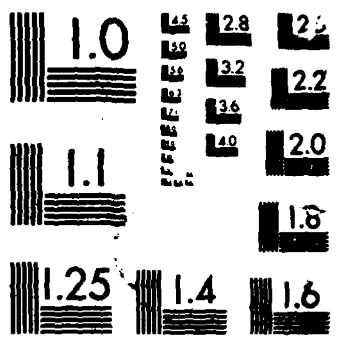
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this work, ~~we have been~~ concerned ~~with~~ various types of failure mechanisms arising due to large strains in solid materials. Specifically, ~~we have~~ addressed issues related to loss of stability, shear localization, void nucleation, stress concentrations and crack-tip deformations. The foregoing failure processes have been investigated analytically within the contexts of finite elasticity and finite strain plasticity theories. The inherently nonlinear phenomena examined in this research program are of crucial importance for understanding the mechanical behavior of solid materials subjected to the increasing demands of military technology. Keywords,

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THEORETICAL INVESTIGATIONS OF SHEAR BAND FORMATION IN FINITE ELASTICITY

FINAL REPORT

C.O. Horgan and R. Abeyaratne

November 15, 1986

U.S. Army Research Office

Contract DAAG 29-83-K-0145

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Abstract

In this work, we have been concerned with various types of failure mechanisms arising due to large strains in solid materials. Specifically, we have addressed issues related to loss of stability, shear localization, void nucleation, stress concentrations and crack-tip deformations. The foregoing failure processes have been investigated analytically within the contexts of finite elasticity and finite strain plasticity theories. The inherently nonlinear phenomena examined in this research program are of crucial importance for understanding the mechanical behavior of solid materials subjected to the increasing demands of military technology.

1. Introduction:

In large deformations of ductile solids, a commonly observed phenomenon involves the sudden transition of a smoothly varying deformation field to one with narrow bands of highly localized shear deformation. The formation of such shear bands often leads to fracture along these zones of intense shearing, so that localization serves as a trigger for rupture. The appearance of Lüder's bands in metals is a well-known example of this.

Shear bands are often seen to emerge at points of material or load discontinuity under large loadings: for example, at voids, inclusions and crack tips or in impact and penetration problems. Thus the understanding and analysis of shear band formation and growth is clearly pertinent to Army concerns.

Considerable efforts have been made recently towards a theoretical study of this inherently nonlinear phenomenon. The process of localization is viewed as a bifurcation caused by a material instability. Such a bifurcation is possible when the mathematical character of the governing differential equations ceases to be elliptic. Often, such a loss of ellipticity leads to the emergence of solutions which are not "smooth."

In the work carried out under this contract, we have been concerned with various types of failure mechanisms arising due to large strains in solid materials. Specifically, we have addressed issues related to loss of stability, shear localization, void nucleation, stress concentrations and crack-tip deformations. The foregoing failure processes have been investigated analytically within the contexts of finite elasticity and finite strain plasticity theories. Clearly a thorough understanding of these phenomena is of vital Army concern.

Specific problems treated have included: (i) initiation of localized shear and localized surface bifurcations at a void in an infinite compressible medium under pressure load conditions [1] (ii) pressure maxima and localized shear bifurcations for internally pressurized circular tubes and spherical shells [2] (iii) void nucleation and associated bifurcation phenomena; [3,4] (iv) shear localization at the tip of a crack loaded in Mode I. [5] (v) stress concentration problems in finite anti-plane shear [6], (vi) dissipative effects at a surface of strain discontinuity [7].

2. (a), (b) Description of Main Results:

(i) Initiation of localized plane deformations:

The problem first investigated was concerned with localized deformations at a void. Conditions for the initiation of both localized shear and localized surface bifurcations at a circular cavity under plane strain conditions were obtained. The cases of a pressurized cavity and a free cavity under remotely applied pressure loading were both considered. The analysis was carried out for a class of foam rubber materials which are compressible and elastic. The results are obtained in exact closed analytic form. Figures 1 and 2 illustrate some of the results and are taken from [1].

(ii) Material instabilities of pressurized elastic cylinders and spheres:

When hollow cylinders or spheres are subjected to pressure loading, they usually fail by one of two mechanisms, viz. by unrestrained growth or by shearing instability. Often, a graph of the applied pressure vs. the deformed radius first rises to a maximum and then decreases. If the applied pressure reaches this maximum value, unrestrained growth takes place. However, depending on the particular material and the geometry, a shear instability might precede this. In the case of incompressible materials this phenomenon has been studied by others. The effect of compressibility does not seem to have been considered. We have examined this in the case of a particular foam rubber material. A closed form analytical solution was found and the precise conditions for the onset of instability were determined [2].

It was found that when the ratio of the outer undeformed radius to the inner undeformed radius is larger than a certain critical value the shear bifurcation occurs before the pressure maximum is attained. When this ratio is smaller than this critical value, the converse is true. Figure 3 illustrates some of the results for a pressurized cylinder and is taken from [2].

(iii) Void nucleation and associated bifurcation phenomena:

A common failure mode observed in metals is the appearance of voids, which then coalesce, to form cracks. Thus the nucleation of voids (usually

at a pre-existing imperfection such as a micro-void) is a pre-cursor to rupture. Infinitesimal theories of solid mechanics (including the classical theories of small strain plasticity) do not predict this phenomenon. The usual procedure is to impose some ad hoc criterion to signal the nucleation of the void. We have shown that this is not necessary in a fully nonlinear theory. The nucleation of a void is predicted naturally, as a bifurcation. The critical load is predicted automatically by the theory and does not need to be prescribed in advance. We examined this issue first in the case of a nonlinearly elastic material [3] and then in the case of large strain plasticity [4].

In [3], we have carried out an explicit analysis of nucleation of a void from a pre-existing micro-void within the frame work of nonlinear elasticity theory. The problem considered concerns a hollow circular cylinder composed of a particular compressible material, the Blatz-Ko material. The outer boundary is subjected to a prescribed radial stretch λ while the cavity surface remains free of traction. A closed-form solution to this plane strain problem is obtained. Attention is focused on the growth of the cavity in the particular case in which its undeformed radius is small. It is found that first, the cavity radius increases slowly as the applied stretch is increased until it reaches a certain critical value λ_{cr}. Rapid growth suddenly takes place beyond this point (see Fig. 4 here, taken from [3]). Of course, such behavior is not predicted by linear elasticity theory. The limiting case of a micro-void is considered in detail. It is shown that the deformation field in this case coincides with that obtained from the analysis of a bifurcation problem for a solid cylinder (see the solid curve in Fig. 4). Such mathematical bifurcation problems have been the subject of much recent investigation.

In [4] we have examined the analogous bifurcation problem arising in finite strain plasticity (a finite strain version of J_2-flow theory) for a solid sphere subjected to a monotonically increasing radial tensile dead load at its outer boundary. Again a critical load at which void nucleation can occur is obtained. The relation between applied load and cavity radius for subsequent cavity growth is also established. It is shown in [4] that the bifurcation arising is inherently associated with the kinematic

nonlinearity: the classical infinitesimal strain theory of plasticity does not exhibit such a bifurcation.

Several important unanswered questions arise in studies of void nucleation and growth in nonlinear solids. One of the authors (C.O.H.) has proposed to continue such investigations under ARO support in a proposal entitled "Theoretical Investigations of Void Nucleation and Growth in Nonlinear Solids," (by C.O. Horgan and T.J. Pence) submitted to ARO on November 4, 1986.

(iv) Localization of deformation near a crack-tip:

In this investigation [5] the crack-tip fields for a plane strain mode I crack were examined. The surrounding material was modeled as being incompressible, isotropic, and elastic, and was characterized through its response function in shear, which in turn was taken to be of a power-law softening form at large strains. The analysis predicts the presence of two localized shear discontinuities emanating from each crack-tip. The stress and deformation fields in the various zones around the crack-tip were also determined.

(v) Nonlinearity and stress concentrations:

In previous work, the authors have developed an analytical approach for obtaining bounds on elastic stress concentration factors in the theory of finite anti-plane shear of homogeneous isotropic incompressible materials. For the problem of an infinite slab, with a traction-free circular or elliptical cavity, subject to a state of finite simple shear deformation, explicit estimates for the stress concentration factor were obtained in terms of the cavity geometry, applied stress at infinity and constitutive parameters. In [6] numerical results for these stress concentration factors have been obtained using a finite-difference scheme which confirm the accuracy of the analytical estimates.

(vi) Dissipative Effects at a surface of strain discontinuity

This study [7] has been concerned with dissipative effects at a surface of strain discontinuity. In view of the discontinuity of the deformation gradient field at such a surface, quasi-static motions of even an elastic body involving such fields, has a dissipation of mechanical energy

associated with it. A model problem has been studied using an internal variable framework, together with notions of maximum dissipation. The resulting behavior of the nominally elastic body was shown to be hysteretic and reminiscent, in some ways, of elastic-plastic behavior.

(c) <u>List of Publications</u>

[1] R. Abeyaratne and C.O. Horgan, Initiation of localized plane deformations at a circular cavity in an infinite compressible nonlinearly elastic medium. <u>J. Elasticity</u>, <u>15</u>, (1985) 243-256.

[2] D.-T. Chung, C.O. Horgan and R. Abeyaratne, The finite deformation of internally pressurized hollow cylinders and spheres for a class of compressible elastic materials. <u>Int. J. Solids and Structures</u>, <u>22</u> (1986) (in press).

[3] C.O. Horgan and R. Abeyaratne, A bifurcation problem for a compressible nonlinearly elastic medium: growth of a micro-void. <u>J. Elasticity</u>, <u>16</u> (1986) 189-200.

[4] D.-T. Chung, C.O. Horgan and R. Abeyaratne, A note on a bifurcation problem in finite plasticity related to void nucleation. <u>Int. J. Solids and Structures</u>, <u>23</u> (1987) (in press).

[5] R. Abeyaratne and J.-S. Yang, Localized shear discontinuities near the tip of a mode I crack, <u>J. Elasticity</u>, <u>17</u> (1987) (in press).

[6] C.O. Horgan and S.A. Silling, Stress concentration factors in finite anti-plane shear: numerical calculations and analytical estimates. <u>J. Elasticity</u>, <u>17</u> (1987) (in press).

[7] R. Abeyaratne and J.K. Knowles, Nonelliptic elastic materials and the modeling of dissipative mechanical behavior: an example. <u>J. Elasticity</u> (in press).

(d) <u>Scientific Personnel and Degrees Awarded</u>:

(i) J.S. Yang (Ph.D. awarded December 1985)

Dissertation Title: Some nonlinear aspects of crack-tip fields in finite elasticity.

(ii) D.-T. Chung (Ph.D. awarded August 1986)

Dissertation Title: The deformation and stability of a pressurized circular tube and spherical shell in finite elasticity and finite plasticity.

List of Figures

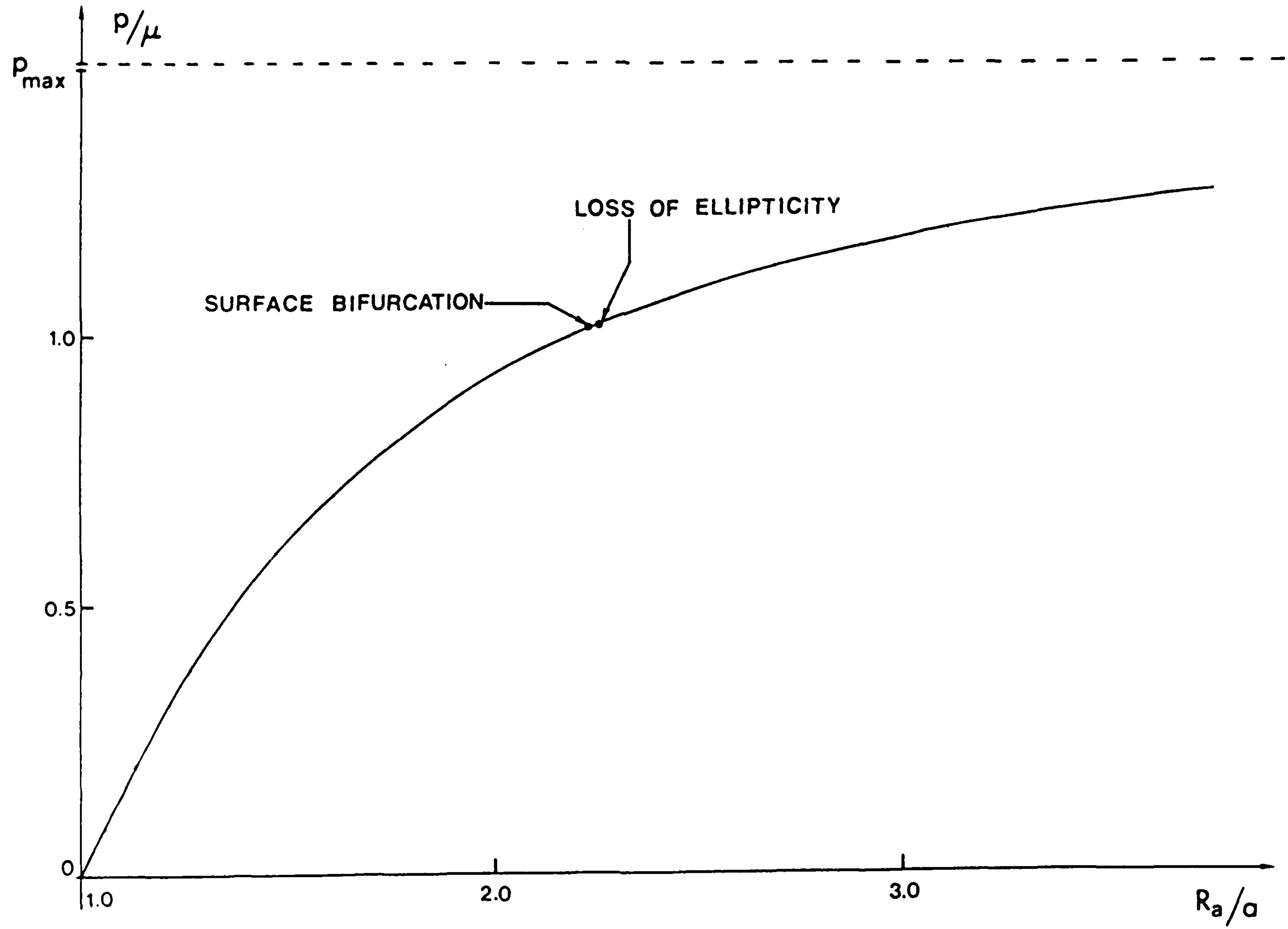

Figure 1. Deformed cavity radius R_a versus applied internal pressure

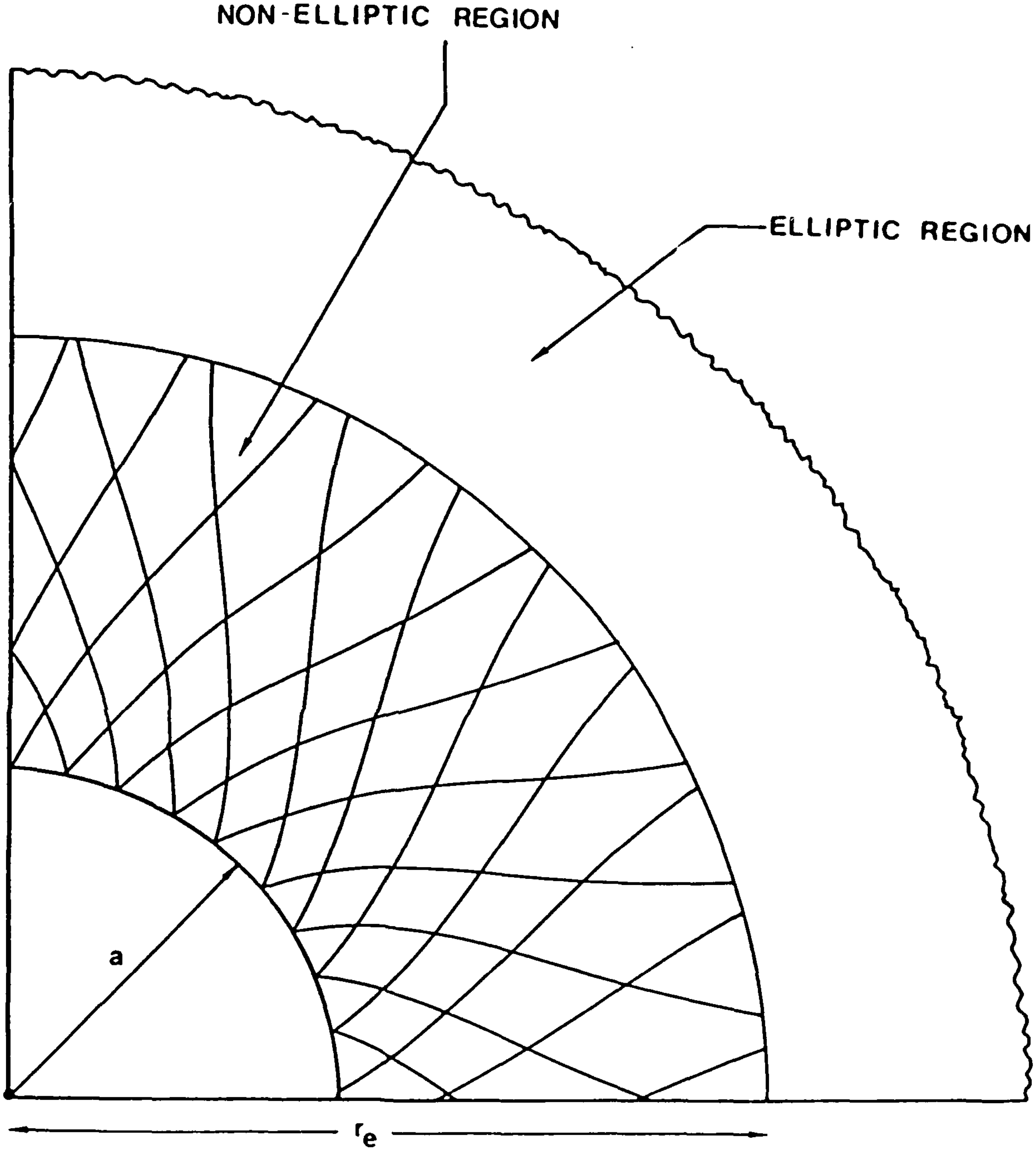

Figure 2. Elliptic and non-elliptic regions with families of material characteristic curves defined by $(4.13)_1$; internal pressure = 1.32862μ.

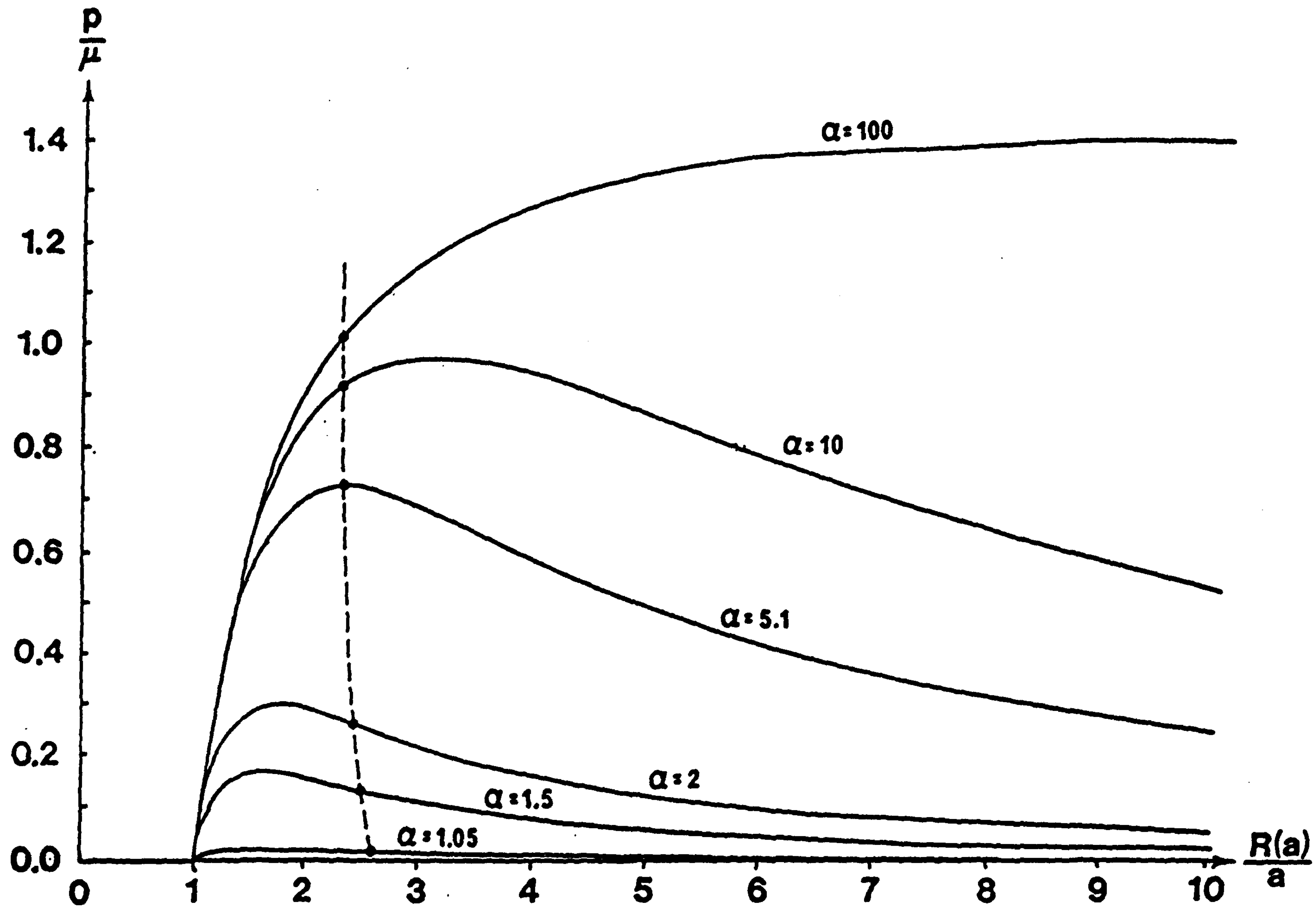

Figure 3. Applied pressure versus ratio of deformed to undeformed radius for hollow cylinders for different radius ratios $\alpha = b/a$. The dotted curve connects the points $(p_e/\mu, R(a)/a)$, where p_e denotes the pressure at which ellipticity is lost.

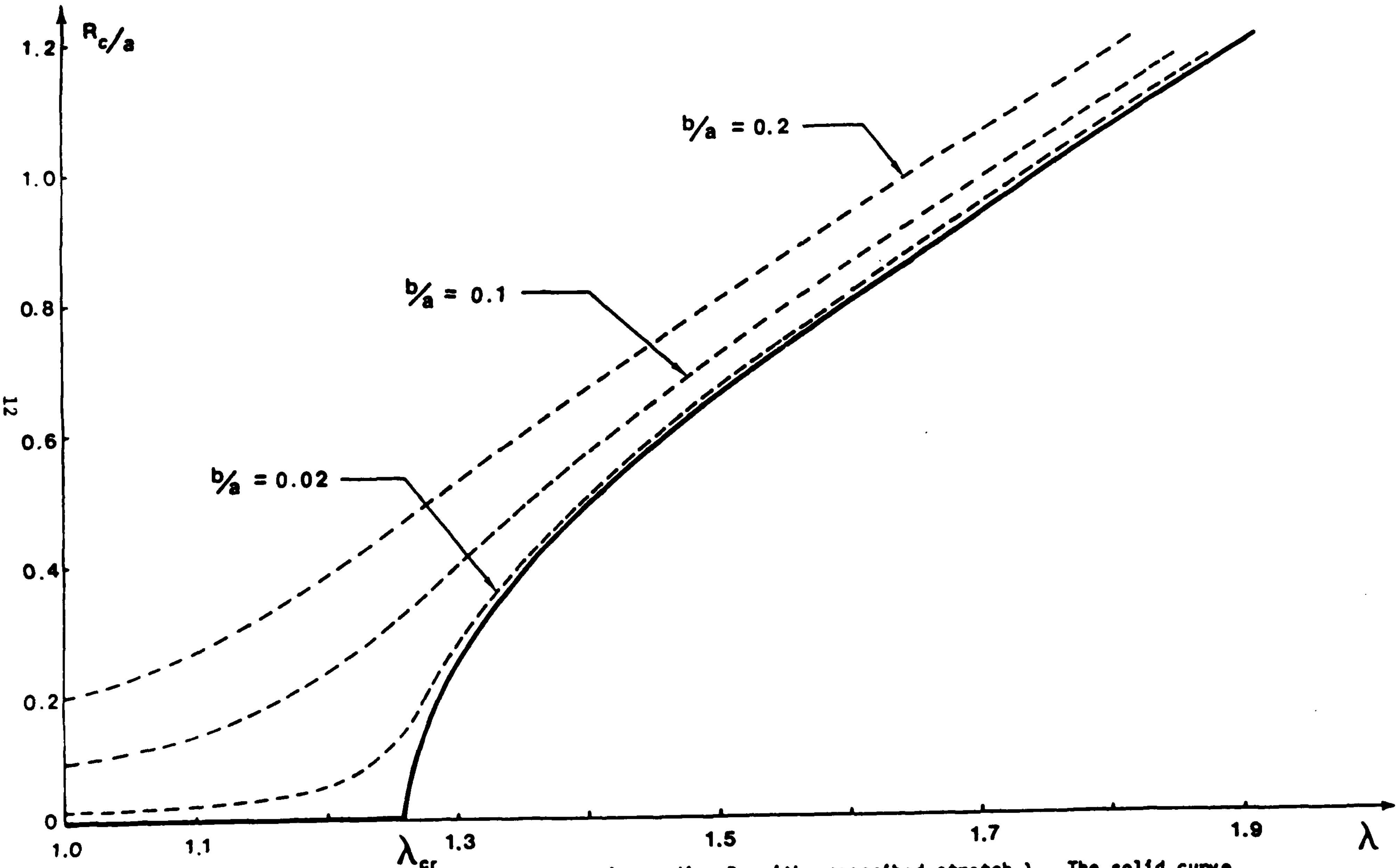

Figure 4. Variation of the deformed cavity radius R_c with prescribed stretch λ. The solid curve pertains to the bifurcation of a solid cylinder (Section 2) while the dashed curves describe the growth of a pre-existing void for different undeformed void radii b (Section 3). The solid curve also describes the limiting case of a micro-void ($b \to 0+$).

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